



University of Groningen

Transient quintessence from group manifold reductions or how all roads lead to Rome

Bergshoeff, E; Collinucci, A; Gran, U; Nielsen, M; Roest, D

Published in:
Classical and Quantum Gravity

DOI:
[10.1088/0264-9381/21/8/003](https://doi.org/10.1088/0264-9381/21/8/003)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2004

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Bergshoeff, E., Collinucci, A., Gran, U., Nielsen, M., & Roest, D. (2004). Transient quintessence from group manifold reductions or how all roads lead to Rome. *Classical and Quantum Gravity*, 21(8), 1947-1969. [PII S0264-9381(04)74508-6]. <https://doi.org/10.1088/0264-9381/21/8/003>

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Transient quintessence from group manifold reductions or how all roads lead to Rome

E Bergshoeff¹, A Collinucci¹, U Gran², M Nielsen¹ and D Roest¹

¹ Centre for Theoretical Physics, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands

² Department of Mathematics, King's College London, Strand, London WC2R 2LS, UK

E-mail: e.a.bergshoeff@phys.rug.nl, a.collinucci@phys.rug.nl, m.nielsen@phys.rug.nl, d.roest@phys.rug.nl and ugran@math.kcl.ac.uk

Received 9 January 2004

Published 16 March 2004

Online at stacks.iop.org/CQG/21/1947 (DOI: 10.1088/0264-9381/21/8/003)

Abstract

We investigate the accelerating phases of cosmologies supported by a metric, scalars and a single exponential scalar potential. The different solutions can be represented by trajectories on a sphere and we find that quintessence happens within the ‘arctic circle’ of the sphere. Furthermore, we obtain multi-exponential potentials from 3D group manifold reductions of gravity, implying that such potentials can be embedded in gauged supergravities with an M-theory origin. We relate the double exponential case to flux compactifications on maximally symmetric spaces and S-branes. In the triple exponential case our analysis suggests the existence of two exotic $S(D-3)$ -branes in D dimensions.

PACS numbers: 04.65.+e, 11.25.Yb

1. Introduction

Since the observational evidence for an accelerating universe and a nonzero cosmological constant [1, 2], there has been a growing interest in finding (stable or unstable) de Sitter solutions [3–7] or more general accelerating cosmologies from M-theory [8–25] (for general results on accelerating cosmologies, i.e. earlier and/or without an M-theory origin, see [26–42]).

One of the purposes of this paper is to investigate the possibility of transient acceleration, i.e. a period of quintessence³, for a large class of cosmologies whose solutions are described by a metric and N scalars. We assume that the scalar potential is given by a single exponential. This has the consequence that effectively the scalar potential depends on only one scalar. All other $N-1$ scalars are represented by their kinetic terms only. Since the metric cannot

³ Quintessence corresponds to the universe being filled with a perfect fluid with equation of state $p = \kappa\rho$, where $-1 \leq \kappa < -1/3$. This induces acceleration.

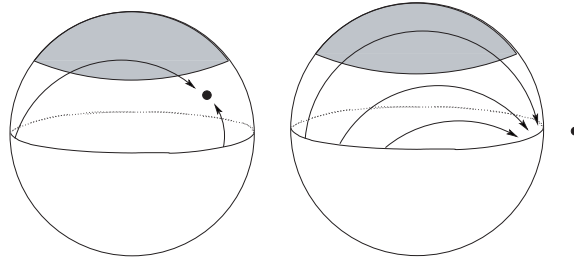


Figure 1. Each cosmological solution is represented by a curve on the sphere. In the left-hand figure ‘Rome’, represented by the dot, is on the sphere and each curve is directed from the equator towards ‘Rome’, which corresponds to a power-law solution to the equations of motion. In the right-hand figure ‘Rome’ is not on the sphere and each curve, again being directed towards ‘Rome’, begins and ends on the equator. In this case ‘Rome’ is not a solution. The accelerated expansion of the solution happens whenever the curve lies within the ‘arctic circle’. This region is given by the shaded area.

distinguish between these different $N - 1$ scalars, there is no qualitative difference between the $N = 2$ scalar cosmology and the $N > 2$ scalar cosmologies. We therefore only consider the one-scalar ($N = 1$) and two-scalar ($N = 2$) cosmologies.

The cosmological solutions discussed in this paper have been given some time ago [40, 43]. The fact that these cosmologies, for particular cases at least, exhibit a period of quintessence was noted recently in [11], where a specific class of solutions was obtained by compactification over a compact hyperbolic space (for earlier discussions, see [30–32, 40, 44]). The relation with S-branes was subsequently noted in [12, 13, 16] (for general literature on S-brane solutions, see [14, 15, 20, 30, 32, 45–52]).

In this work we will discuss systematically the accelerating phases of all two-scalar cosmologies with a single exponential potential by associating with each solution a trajectory on a two-sphere. It turns out that all trajectories have the property that, when projected onto the equatorial plane, they reduce to straight lines which are directed towards a point that we will call ‘Rome’. Depending on the specific dilaton coupling of the potential, this point can either be on the sphere or not. In the former case, it corresponds to a power-law solution for the scale factor, whereas in the latter case, it is not a solution. We find that the accelerating phase of a solution is represented by the part of the trajectory that lies within the ‘arctic circle’ on the sphere (see figure 1). This enables us to calculate the expansion factors in a straightforward way for each of the solutions.

In the second part of this work we show the higher-dimensional origin of a class of solutions. Applying 3D group manifold reductions we consider the embedding in gauged supergravities with an M-theory origin. The higher-dimensional origin of certain power-law solutions is a (locally) Minkowskian spacetime in $D \geq 6$ dimensions [49, 51, 53].

We extend our analysis to n -tuple exponential potentials with $n = 2, 3$ and 6 that follow from the group manifold reduction. In the case of double exponential potentials we compare our results with those that follow from the flux compactification on a maximally symmetric space and S-branes. In the triple exponential case our analysis suggests the existence of two exotic $S(D - 3)$ -branes in D dimensions which have not been constructed before in the literature.

This paper is organized as follows. In sections 2.1–2.3 we present, under the assumptions stated, the most general N -scalar accelerating cosmology in four dimensions. The accelerating phases of these cosmologies are discussed in section 2.4. Their equations of state and the one-scalar truncations are discussed in sections 2.5 and 2.6, respectively. In section 3 we

discuss the higher-dimensional origin of a class of $D = 4$ solutions. In section 3.1 we explain the group manifold reduction and in section 3.2 we consider the special case of a single exponential potential. In sections 3.3, 3.4 we consider the case of double and triple exponential potentials and relate to other reductions and S-branes. The embedding into M-theory and gauged maximal supergravities is discussed in section 3.5. Finally, in section 4 we comment on inflation, the compactness of the group manifolds and exotic S-branes.

2. Accelerating cosmologies in four dimensions

2.1. Multi-scalar gravity with a single exponential potential

Our starting point is gravity coupled to N scalars [36] which we denote by $(\varphi, \vec{\phi})$. We assume that the scalar potential consists of a single exponential term:

$$\mathcal{L} = \sqrt{-g} \left[R - \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}(\partial\vec{\phi})^2 - V(\varphi, \vec{\phi}) \right], \quad V(\varphi, \vec{\phi}) = \Lambda \exp(-\alpha\varphi - \vec{\beta} \cdot \vec{\phi}), \quad (1)$$

where we restrict⁴ to $\Lambda > 0$. To characterize the potential we introduce the following parameter:

$$\Delta \equiv \alpha^2 + |\vec{\beta}|^2 - \frac{2(D-1)}{(D-2)} = \alpha^2 + |\vec{\beta}|^2 - 3 \quad \text{for } D = 4. \quad (2)$$

This parameter, first introduced in [54], is invariant under toroidal reductions.

The kinetic terms of the dilatons are invariant under $SO(N)$ -rotations of $(\varphi, \vec{\phi})$. However, in the scalar potential the coefficients α and $\vec{\beta}$ single out one direction in N -dimensional space. Therefore the Lagrangian (1) is only invariant under $SO(N-1)$. The remaining generators of $SO(N)$ can be used to set $\vec{\beta} = 0$, in which case only the scalar φ appears in the scalar potential. Such a choice of basis leaves Δ invariant.

Motivated by observational evidence, we choose a flat FLRW Ansatz. This basically means a spatially flat metric that can only contain time-dependent functions. One can always perform a reparametrization of time to bring the metric to the following form:

$$ds^2 = -a(u)^{2\delta} du^2 + a(u)^2 dx_3^2, \quad (3)$$

for some δ . In this paper we will choose δ as follows⁵:

$$\text{cosmic time:} \quad \delta = 0, \quad u = \tau, \quad \frac{da}{d\tau} = \dot{a}, \quad (4)$$

$$\text{non-cosmic time:} \quad \delta = 3, \quad u = t, \quad \frac{da}{dt} = a'. \quad (5)$$

As a part of the Ansatz, we also assume

$$\varphi = \varphi(u), \quad \vec{\phi} = \vec{\phi}(u). \quad (6)$$

For this Ansatz one can reduce the $N-1$ scalars $\vec{\phi}$ that do not appear in the potential to one scalar by using their field equations as follows:

$$\frac{d^2 \vec{\phi}}{du^2} = (\delta - 3) \frac{d \log a}{du} \frac{d \vec{\phi}}{du} \Rightarrow \frac{d \vec{\phi}}{du} = \vec{c} a^{\delta-3}, \quad (7)$$

⁴ We make this choice in order to obtain quintessence and therefore accelerating solutions.

⁵ The non-cosmic time corresponds to the gauge in which the lapse function $N \equiv \sqrt{-g_{tt}}$ is equal to the square root of the determinant γ of the spatial metric, i.e. $N = \sqrt{\gamma}$, whereas cosmic time corresponds to $N = 1$. We thank Marc Henneaux for a discussion on this point.

where \vec{c} is some constant vector. The only influence of the $N - 1$ scalars comes from their total kinetic term:

$$\left| \frac{d\vec{\phi}}{du} \right|^2 = |\vec{c}|^2 a^{2\delta-6}. \quad (8)$$

Therefore, from the metric point of view, there is no difference between $N = 2$ and $N > 2$ scalars (under the restriction of a single exponential potential). The truncation of the system (1) to one scalar corresponds to setting $\vec{c} = 0$.

To summarize, we will be using the following Lagrangian:

$$\mathcal{L} = \sqrt{-g} \left[R - \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}(\partial\phi)^2 - V(\varphi) \right], \quad V(\varphi) = \Lambda \exp(-\alpha\varphi), \quad (9)$$

with $\Lambda > 0$ and we choose the convention $\alpha \geq 0$. From now on we will use $\Delta = \alpha^2 - 3$ instead of α .

In the next two subsections we will first discuss the critical points corresponding to the system (9) and then the solutions that interpolate between these critical points. We will use cosmic time (4) when discussing the critical points in section 2.2 and non-cosmic time (5) when dealing with the interpolating solutions in section 2.3.

2.2. Critical points

It is convenient to choose a basis for the fields, such that they parametrize a two-sphere. In this basis we will find that all constant configurations (critical points) correspond to power-law solutions for the scale factor $a(\tau) \sim \tau^p$ for some p . By studying the stability of these critical points [27, 55] one can deduce that there exist interpolating solutions which tend to these points in the far past or the distant future. We will actually be able to draw these interpolating solutions without having to do any stability analysis.

We begin by choosing the flat FLRW Ansatz (3) in cosmic time:

$$ds^2 = -d\tau^2 + a(\tau)^2(dx^2 + dy^2 + dz^2). \quad (10)$$

The Einstein equations for the system (9) with this Ansatz become

$$H^2 = \frac{1}{12}(\dot{\phi}^2 + \dot{\phi}^2) + \frac{1}{6}V, \quad (11)$$

$$\dot{H} = -\frac{1}{4}(\dot{\phi}^2 + \dot{\phi}^2), \quad (12)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter and the dot denotes differentiation w.r.t. τ . Equations (11) and (12) are usually referred to as the Friedmann equation and the acceleration equation, respectively. The scalar equations are

$$\ddot{\phi} = -3H\dot{\phi} + \sqrt{\Delta + 3}V, \quad \ddot{\phi} = -3H\dot{\phi}. \quad (13)$$

We define the following three variables:

$$x = \frac{\dot{\phi}}{\sqrt{12}H}, \quad y = \frac{\dot{\phi}}{\sqrt{12}H}, \quad z = \frac{\sqrt{V}}{\sqrt{6}H}. \quad (14)$$

In these variables the Friedmann equation (11) becomes the defining equation of a two-sphere [42, 55]

$$x^2 + y^2 + z^2 = 1. \quad (15)$$

This means that we can think of solutions as points or trajectories on a globe. It turns out that cosmological solutions are either eternally expanding (i.e. $H > 0$) or eternally contracting ($H < 0$), but cannot have an expanding phase and then a contracting phase (or vice versa).

Since we are only interested in expanding universes, we will only be concerned with the upper hemisphere (i.e. $z > 0$). In terms of x and y the scalar equations become

$$\frac{\dot{x}}{H} = -3z^2(x - \sqrt{1 + \Delta/3}), \quad (16)$$

$$\frac{\dot{y}}{H} = -3z^2y. \quad (17)$$

We can rewrite the acceleration equation (12) as follows:

$$\frac{\dot{H}}{H^2} = -3(x^2 + y^2). \quad (18)$$

If we now solve for the critical points ($\dot{x} = 0, \dot{y} = 0$), we can then integrate (18) twice and obtain the following power-law solutions for $a(\tau)$ [36]:

$$a(\tau) \sim \tau^p, \quad \text{where} \quad p = \frac{1}{3(x_c^2 + y_c^2)}, \quad (19)$$

and the following solutions for the scalars:

$$\varphi = \sqrt{12}p x_c \log(\tau) + \text{constant}. \quad (20)$$

We thus find the following critical points:

- The equator:

$$z = 0, \quad x^2 + y^2 = 1. \quad (21)$$

Every point on the equator of the sphere is a critical point with power-law behaviour $a \sim \tau^{1/3}$.

- ‘Rome’:

$$x = \sqrt{1 + \Delta/3}, \quad y = 0, \quad z = \sqrt{-\Delta/3}. \quad (22)$$

This critical point yields a power-law behaviour of the form (we ignore here irrelevant constants that rescale time)

$$a \sim \tau^{1/(\Delta+3)} \quad \text{for} \quad -3 < \Delta < 0, \quad a \sim e^\tau \quad \text{for} \quad \Delta = -3. \quad (23)$$

Note that the greater Δ is, the further ‘Rome’ gets pushed towards the equator, and for $\Delta = 0$ it is on the equator.

Although the equatorial points (a.k.a. kinetic-dominated solutions) do solve (15)–(18) as critical points, they are not proper solutions of (11)–(13) in terms of the fundamental fields, since $z = 0$ would imply that $V = 0$, which is impossible for $\Lambda \neq 0$ unless φ is infinite at all times. However, these points will be interesting to us, as they will give information about the asymptotics of the interpolating solutions.

In contrast to the equator, the ‘Rome’ critical point is a physically acceptable solution of the system, provided it is well defined on the globe (i.e. $\Delta < 0$). In the case where $\Delta = -3$ it becomes de Sitter (i.e. $a \sim e^\tau$), as one would expect, since $V = \Lambda$.

Besides these critical points there are other solutions, which are not points but rather trajectories. In fact, we can already determine their shapes. Dividing (16) and (17) we obtain the following:

$$\frac{dy}{dx} = \frac{y}{x - \sqrt{1 + \Delta/3}}. \quad (24)$$

Integrating this we get the following relation between x and y :

$$y = C(x - \sqrt{1 + \Delta/3}), \quad (25)$$

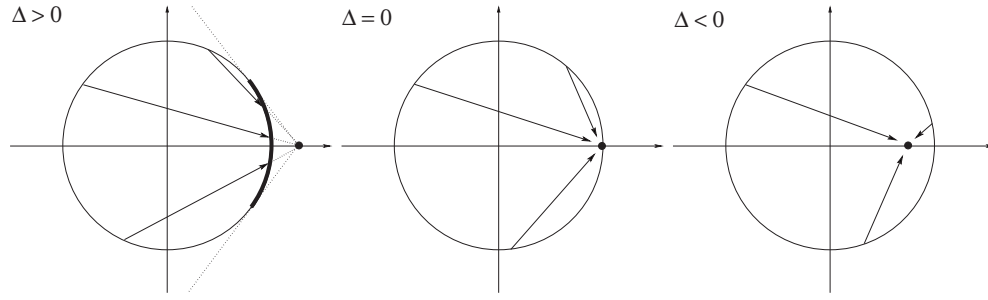


Figure 2. The solutions represented as straight lines in the (x, y) -plane for $\Delta > 0$ where ‘Rome’ is not on the sphere, $\Delta = 0$ where ‘Rome’ is on the equator and $-3 \leq \Delta < 0$ where ‘Rome’ is on the sphere. The thick arc in the left-hand figure represents the attracting portion of the equator [38].

where C is an arbitrary constant⁶. This relation tells us that if we project the upper hemisphere onto the equatorial plane, in other words, if we view the sphere from above, any solution to the equations of motion must trace out a straight line that lies within the circle defined by $x^2 + y^2 = 1$ and has a y -intercept at $(x = \sqrt{1 + \Delta/3}, y = 0)$. From now on, we will refer to that point as ‘Rome’⁷. Note that all lines intersect at ‘Rome’, independently of whether it is on the globe ($\Delta < 0$), right on the equator ($\Delta = 0$) or off the globe ($\Delta > 0$). These lines can only have critical points as end-points. So each line is a solution, which interpolates between two power-law solutions. In a similar, yet physically inequivalent context, such a line was found in [33].

Now that we know the shapes of the trajectories, let us figure out their time orientations. By looking at (16) we realize that the time derivative of x is positive when $x < \sqrt{1 + \Delta/3}$ and negative when $x > \sqrt{1 + \Delta/3}$. This tells us that *all roads lead to Rome*. Figure 2 illustrates this for the cases where ‘Rome’ is off the globe, right on the equator or on the globe.

One can also determine the orientations of the trajectories by analysing the stability of the critical points. One will find that whenever ‘Rome’ is on the globe (i.e. $\Delta = 0$ and $\Delta < 0$), it is stable (i.e. an attractor), and the points on the equator are all unstable (i.e. repellers), except for ‘Rome’ when $\Delta = 0$. In the case where ‘Rome’ is off the globe (i.e. $\Delta > 0$), the equator splits up into a repelling and an attracting region. The attracting region turns out to be the portion of the equator that can ‘see’ ‘Rome’. In other words, any point on the equator that can be joined to ‘Rome’ by a straight line such that the line does not intersect the equator again before reaching ‘Rome’ is attracting. To summarize, for $\Delta > 0$, all points on the equator with $x > \sqrt{3/(\Delta + 3)}$ are attracting, and the rest are repelling. In the first illustration of figure 2, the attracting portion of the equator is depicted by the thick arc.

2.3. Interpolating solutions

To solve the equations of motion, it is convenient to use the FLRW Ansatz (3) in noncosmic time:

$$ds^2 = -a(t)^6 dt^2 + a(t)^2 dx_3^2. \quad (26)$$

⁶ Since C is finite one might think that this excludes the line defined by $x = \sqrt{1 + \Delta/3}$. However, that line can be obtained by taking the inverse of (24) and solving for x as a function of y .

⁷ Note that we have extended our definition of ‘Rome’: only if ‘Rome’ is on the globe ($\Delta < 0$) is it equal to the critical point discussed before.

Substituting this Ansatz into the Einstein equations yields

$$F^2 = \frac{1}{3}F' + \frac{1}{12}(\phi'^2 + \varphi'^2), \quad (27)$$

$$F' = \frac{1}{2}Va^6, \quad (28)$$

where $F = a'/a$ is a Hubble parameter-like function, and the prime denotes differentiation w.r.t. t . The equations for the scalars are

$$\phi'' = 0, \quad \varphi'' = \sqrt{\Delta + 3}Va^6. \quad (29)$$

Combining (29) and (28) gives the following solutions for the scalars:

$$\varphi = 2\sqrt{\Delta + 3} \log(a) + a_1 t + b_1, \quad \phi = a_2 t + b_2. \quad (30)$$

By substituting this into equation (25) we can deduce that the slope of the line is given by $C = a_2/a_1$. Substituting the scalars into (27) and (28) we are now left with the following two equations:

$$F' = -\Delta F^2 - \sqrt{\Delta + 3}a_1 F - \frac{1}{4}(a_1^2 + a_2^2) \quad (31)$$

$$= \frac{1}{2}\Lambda e^{-\sqrt{\Delta+3}(b_1+a_1 t)} a^{-2\Delta}. \quad (32)$$

Keeping in mind that F' must be positive due to (32) we can now solve for F in the three different cases where Δ is positive, zero and negative. We can then easily find $a(t)$. We will choose b_1 (the constant part of φ) such that all solutions for $a(t)$ have a proportionality constant of 1, which does not affect the cosmological properties of the solutions. The integration constants appearing in the solutions are defined as follows:

$$c_1 = \frac{-\sqrt{\Delta + 3}a_1}{2\Delta}, \quad c_2 = \frac{\sqrt{3a_1^2 - \Delta a_2^2}}{2}, \quad d_1 = -\frac{a_1^2 + a_2^2}{4\sqrt{3}a_1}, \quad d_2 = -\sqrt{3}a_1. \quad (33)$$

Below we present the solutions [40, 43] and their late- and early-time asymptotic behaviours (we give the latter without any irrelevant constants that rescale time):

1. $\Delta > 0$:

$$a(t) = e^{c_1 t} \cosh(c_2 t)^{1/\Delta}, \quad \text{for } -\infty < t < +\infty. \quad (34)$$

The positivity of F' requires a_1 to be negative, and it also imposes the following constraint:

$$\left(\frac{a_2}{a_1}\right)^2 < \frac{3}{\Delta}. \quad (35)$$

This solution corresponds to a generic line on the first illustration in figure 2. It starts on the equator somewhere to the left of $x = \sqrt{3/(\Delta + 3)}$, then moves in the direction of ‘Rome’, but ends on the equator on the right-hand side. Note that constraint (35) is simply the requirement that the slope of the line is bounded from above and from below such that the line actually intersects the sphere. We can confirm this asymptotic behaviour of the solution by converting to cosmic time (4) for $t \rightarrow -\infty$ and $t \rightarrow +\infty$ with the relation $a(t)^3 dt = d\tau$:

$$\begin{aligned} t \rightarrow -\infty, & \quad \tau \rightarrow 0, & a \rightarrow e^t \sim \tau^{1/3}, \\ t \rightarrow +\infty, & \quad \tau \rightarrow +\infty, & a \rightarrow e^t \sim \tau^{1/3}. \end{aligned} \quad (36)$$

2. $\Delta = 0$:

$$a(t) = e^{d_1 t} \exp(e^{d_2 t}), \quad \text{for } -\infty < t < +\infty. \quad (37)$$

The positivity of F' requires a_1 to be negative. This corresponds to a line on the second illustration in figure 2. It starts on the equator and reaches ‘Rome’⁸, which is also on the equator. Its asymptotic behaviour goes as follows:

$$\begin{aligned} t \rightarrow -\infty, \quad \tau \rightarrow 0, \quad a \rightarrow e^t \sim \tau^{1/3}, \\ t \rightarrow +\infty, \quad \tau \rightarrow +\infty, \quad a \rightarrow e^{e^t} \sim \tau^{1/3}. \end{aligned} \quad (38)$$

To find the late-time behaviour of a in cosmic time one must realize the following two facts: first, $a(t) \sim \exp(e^t)$ for $t \rightarrow \infty$. Second, in this limit, $a' \sim a$ and therefore a behaves like a normal exponential.

3. $-3 \leq \Delta < 0$:

$$a(t) = e^{c_1 t} \sinh(-c_2 t)^{1/\Delta}, \quad \text{for } -\infty < t < 0. \quad (39)$$

This solution corresponds to any line on the third illustration in figure 2. It starts at any point on the equator and ends at ‘Rome’. This is reflected in the asymptotics as follows

$$\begin{aligned} t \rightarrow -\infty, \quad \tau \rightarrow 0, \quad a \rightarrow e^t \sim \tau^{1/3}, \\ t \rightarrow 0, \quad \tau \rightarrow +\infty, \quad a \rightarrow (-t)^{1/\Delta} \sim \tau^{1/(\Delta+3)} \quad \text{for } \Delta > -3, \\ \sim e^\tau \quad \text{for } \Delta = -3. \end{aligned} \quad (40)$$

There is one more solution for $-3 \leq \Delta < 0$. If we set $a_1 = a_2 = 0$ we find

$$a(t) = (-t)^{1/\Delta} \quad \text{for } -\infty < t < 0. \quad (41)$$

This solution corresponds to the ‘Rome’ solution itself. For $-3 < \Delta < 0$ the conversion to cosmic time is the following:

$$a \sim \tau^{1/(\Delta+3)}. \quad (42)$$

Note, however, that in the case where $\Delta = -3$, the ‘Rome’ solution (41) and therefore the late-time asymptotics of (39) have a different conversion to cosmic time, namely:

$$a \sim (-t)^{1/\Delta} \sim e^\tau, \quad (43)$$

which we recognize as the de Sitter solution, in agreement with the fact that we have $V = \Lambda$.

The interpolating solutions above are given in non-cosmic time, which as mentioned is related to cosmic time by

$$d\tau = a(t)^3 dt. \quad (44)$$

Integrating this equation yields hypergeometric functions for a generic interpolating solution, which we cannot invert to get the scale factor as a function of cosmic time. However, it is possible to get interpolating solutions in cosmic time for negative Δ when the following constraint on the constants holds:

$$\left(\frac{a_2}{a_1}\right)^2 = 12 \frac{\Delta + \frac{9}{4}}{(2\Delta + 3)^2}, \quad (45)$$

⁸ In this case, ‘Rome’ is again attracting, however to see that, one must perform the stability analysis by going to second-order perturbation. The first order vanishes, which means that the interpolating trajectory approaches ‘Rome’ more slowly than in the cases where $\Delta < 0$.

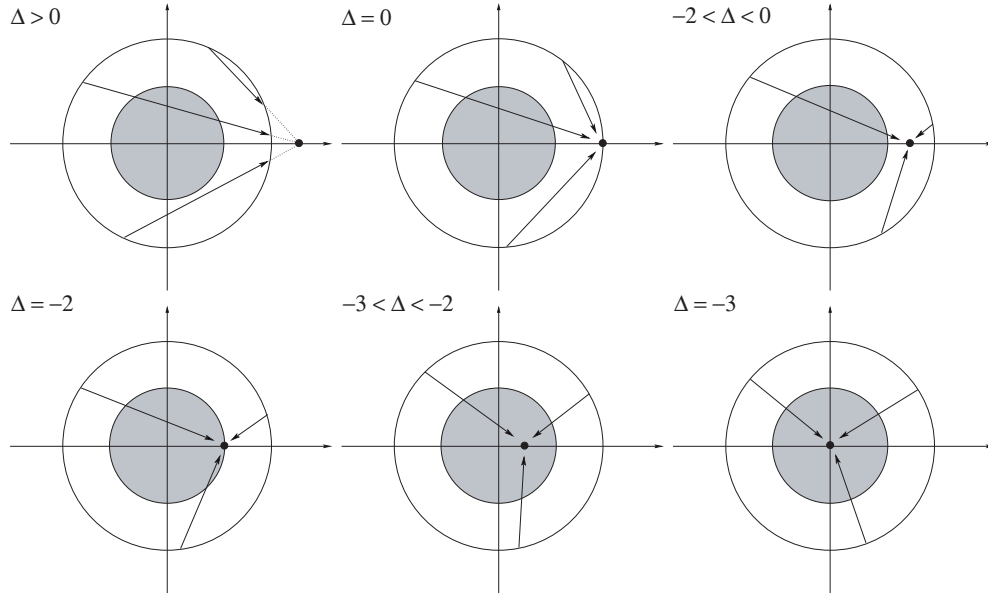


Figure 3. The solutions represented as straight lines in the (x, y) -plane for $\Delta > 0$ where ‘Rome’ is not on the sphere, $\Delta = 0$ where ‘Rome’ is on the equator and $-3 \leq \Delta < 0$ where ‘Rome’ is on the sphere. The inner circle corresponds to the ‘arctic circle’ and solutions are accelerating when they enter the shaded area. The lower part of the figure corresponds to the cases where ‘Rome’ is lying on the ‘arctic circle’, $\Delta = -2$, inside the ‘arctic circle’, $-3 < \Delta < -2$ and on the North Pole, $\Delta = -3$.

which can only be fulfilled for $-9/4 \leq \Delta < 0$. The relation between the two time coordinates is

$$\tau = \frac{2^{-3/\Delta}}{2c_2} \frac{\Delta}{3 + \Delta} (e^{2c_2 t} - 1)^{(3+\Delta)/\Delta}, \quad (46)$$

and the scale factor in cosmic time becomes

$$a(\tau) = (k_1 \tau^{3/(3+\Delta)} + k_2 \tau)^{1/3}, \quad (47)$$

where $k_1 = (2/c_1)^{3/\Delta}$ and $k_2 = k_1 c_1 (2\Delta + 3)/(18 + 6\Delta)$. From this solution, the asymptotic power-law behaviour is easily seen. The special one-scalar case, corresponding to $\Delta = -9/4$, was found in [28].

2.4. Acceleration

In this section we will investigate under which conditions ‘Rome’ and the interpolating solutions represent an accelerating universe, i.e. under which conditions we find quintessence. This can be given a nice pictorial understanding in terms of the two-sphere. We will show that quintessence appears when the trajectory enters the region bounded by an ‘arctic circle’. This is summarized in figure 3.

An accelerating universe is defined by $\ddot{a}/a > 0$. The existence of the ‘arctic circle’ in connection to acceleration can now easily be determined. Assuming an expanding universe and using

$$\frac{\ddot{a}}{a} = \dot{H} + H^2, \quad (48)$$

as well as (18), we see that the condition for acceleration is equivalent to⁹

$$z^2 > \frac{2}{3}, \quad \text{i.e.} \quad x^2 + y^2 < \frac{1}{3}, \quad (49)$$

which exactly yields an ‘arctic circle’ as the boundary of the region of acceleration. The straight line representing the exact solution is parametrized by the constants a_1 and a_2 as found in the previous section. From (49) and (25) it then easily follows that the condition for acceleration leads to the following restriction for the slope of the line:

$$\left(\frac{a_2}{a_1}\right)^2 (2 + \Delta) < 1. \quad (50)$$

This condition is always fulfilled when $\Delta \leq -2$ and otherwise there is an interval of values for a_2^2/a_1^2 yielding an accelerating universe. This can easily be understood from figure 3. In general, a solution will only have transient acceleration. The only exception is when ‘Rome’ lies within or on the ‘arctic circle’, corresponding to $\Delta \leq -2$. Then, from the moment the line crosses the ‘arctic circle’, there will be eternal acceleration [8] towards ‘Rome’. When $\Delta = -2$, there will only be eternal acceleration when ‘Rome’ is approached from the left. The possibilities of acceleration can be summarized as:

- $\Delta > -2$: A phase of transient acceleration is possible.
- $\Delta = -2$: A phase of eternal acceleration is possible.
- $-3 \leq \Delta < -2$: Always a phase of eternal acceleration.

The phase of eternal acceleration can also be understood from the power-law behaviour of the ‘Rome’ solution, i.e. $a(\tau) \propto \tau^{1/(3+\Delta)}$. We have asymptotic acceleration when $1/(3 + \Delta) > 1$, i.e. $\Delta < -2$. In the limiting case $\Delta = -3$, corresponding to ‘Rome’ being on the North Pole, the interpolating solution will asymptote to de Sitter.

2.5. Equation of state

In a cosmological setting, one often writes the matter part of the equations in terms of a perfect fluid, which is described by its pressure p and energy density ρ . These two variables are then assumed to be related via the equation of state:

$$p = \kappa \rho. \quad (51)$$

As is well known in standard cosmology, $\kappa = 0$ corresponds to the matter dominated era, $\kappa = 1/3$ to the radiation dominated era and $\kappa = -1$ to an era dominated by a pure cosmological constant. Quintessence is a generalization of the latter with $-1 \leq \kappa < -1/3$.

In our case, the matter is given by the two scalar fields, and thus p and ρ are given by the difference and sum of the kinetic terms and the potential, respectively:

$$p = \frac{1}{2}(\dot{\varphi}^2 + \dot{\phi}^2) - V, \quad \rho = \frac{1}{2}(\dot{\varphi}^2 + \dot{\phi}^2) + V. \quad (52)$$

Writing the above in terms of x , y and z , we see that the scalars describe a perfect fluid with an equation of state given in terms of the parameter:

$$\kappa = 1 - 2z^2. \quad (53)$$

Hence, κ varies from 1 on the equator to -1 on the North Pole, and we need $\kappa < -1/3$ for quintessence. For the interpolating solutions, which are given as curves on the sphere, κ will depend on time, but it will be constant for the critical points with the following values [8]:

- Equator : $\kappa = 1$,
 - Rome : $\kappa = 1 + \frac{2}{3}\Delta$.
- (54)

⁹ A similar inequality was given in [33] for the one-scalar case, and in terms of the scalars and the potential in [40] for the multi-scalar case.

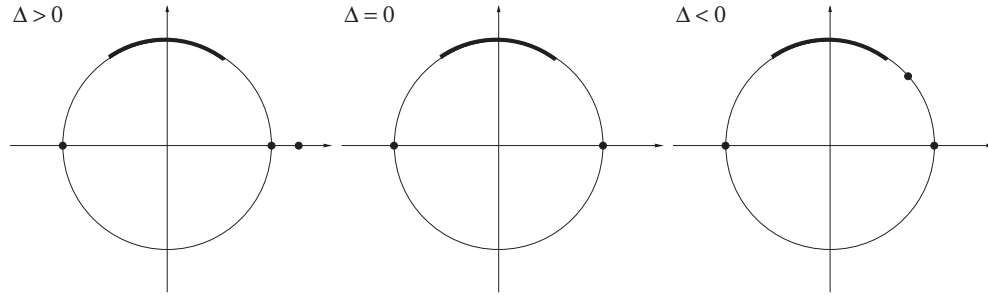


Figure 4. The two-dimensional (x, z) space and the critical points for the one-scalar truncations. The thick curve is the accelerating region. The two points on the x -axis are the equatorial critical points. The third point is ‘Rome’. Note that in the middle illustration ‘Rome’ coincides with the equatorial critical point $x = 1$.

2.6. One-scalar truncations

The analysis has so far been done for two scalars, and as such it also contains the truncation to a system with one scalar with a potential, corresponding to $\phi = 0$. Here we will summarize the results of the previous sections in this truncation. On the sphere this yields $y = 0$, and for the solutions it corresponds to $a_2 = b_2 = 0$.

Since we only have one scalar, the Friedmann equation will define a circle when written in terms of x and z [33, 41]:

$$x^2 + z^2 = 1. \quad (55)$$

The critical points are [27]:

- Equatorial : $z = 0, \quad x^2 = 1,$
- ‘Rome’ ($-3 \leq \Delta < 0$) : $z = \sqrt{-\Delta/3}, \quad x = \sqrt{1 + \Delta/3}.$

The full circle is shown in figure 4, including the critical points, and is just the vertical slice of the two-sphere including the North Pole. The equator therefore becomes two points, and the region bounded by the ‘arctic circle’ now becomes the part of the circle¹⁰ corresponding to $x^2 < 1/3$.

The exact solutions can be obtained from the previous section by setting $a_2 = b_2 = 0$. For $\Delta \geq 0$ the solutions correspond to the curves starting at $x = -1$ and ending at $x = 1$, whereas for $-3 \leq \Delta < 0$ the curves start at either one of the equatorial points and end at ‘Rome’. In all cases where the curve starts at $x = -1$, the corresponding solution will give rise to acceleration. For this reason, interpolating solutions with $\Delta \geq 0$ will always give rise to a period of acceleration. This is in clear contrast to the two-scalar case, where it is possible to avoid acceleration (see figure 3). As for the two-scalar case, if ‘Rome’ lies in the ‘arctic’ region, the solution will be eternally accelerating from the moment it enters this region.

One can also consider the truncation to zero scalars. However, from the scalar field equations, it is seen that this is only consistent if $\Delta = -3$, and this corresponds to the de Sitter solution with $V = \Lambda$.

3. Higher-dimensional origin

In this section we will discuss the higher-dimensional origin of the 4D system (9) by considering group manifold reductions. We will furthermore consider generalizations of

¹⁰ This is equivalent to the accelerating region of [25, 56].

Table 1. The Bianchi classification of the different three-dimensional Lie algebras in terms of components of their structure constants.

Class A	a	(q_1, q_2, q_3)	Algebra	Class B	a	(q_1, q_2, q_3)	Algebra
IX	0	(1, 1, 1)	$so(3)$				
VIII	0	(1, -1, 1)	$so(2, 1)$				
VII ₀	0	(0, 1, 1)	$iso(2)$	VII _{a}	a	(0, 1, 1)	$iso(2)_a$
VI ₀	0	(0, -1, 1)	$iso(1, 1)$	III	1/2	(0, -1, 1)	$iso(1, 1)_{a=1/2}$
				VI _{a}	a	(0, -1, 1)	$iso(1, 1)_a$
II	0	(0, 0, 1)	Heisenberg	IV	1	(0, 0, 1)	Heisenberg $_{a=1}$
I	0	(0, 0, 0)	$u(1)^3$	V	1	(0, 0, 0)	$u(1)^3_{a=1}$

the single exponential potential to n -tuple exponential potentials with $n = 2, 3$ and 6. In particular, we will show that such potentials can be obtained by reduction of 7D gravity over the 3D group manifolds as classified by Bianchi. Furthermore, relations with S-branes will be discussed. Finally, we will comment on the embedding of such potentials in M-theory.

We choose 3D group manifolds since these are well studied and have been classified. Two-dimensional group manifolds only include \mathbb{E}^2 and \mathbb{H}^2 and are implicitly included in our analysis. We anticipate the possibility of a similar analysis for higher-dimensional group manifolds. Our approach is not to be confused with Bianchi cosmology, in which everyday's 3D space is a Bianchi group manifold; we employ the group manifold Ansatz to reduce to 4D, in which we assume a flat FLRW spacetime (3).

3.1. Group manifold reductions

Group manifolds are homogeneous but in principle anisotropic generalizations of maximally symmetric spaces, which are homogeneous and isotropic. Therefore, the dimension of the isometry group $\mathcal{G}^{\text{isom}}$, from which the group manifold takes its name, in general equals the dimension of the manifold. Only in special cases the manifold has more isometries and can even be maximally symmetric. In this section we will take the universal covering of the group manifolds (except for explicit factors of S^1) and will not pay attention to global issues such as compactness; we will comment on this in section 4.

As was proved by Scherk and Schwarz [57], one can always dimensionally reduce a theory containing gravity over a group manifold. This leads to a lower-dimensional theory with a potential and a non-trivial gauge group \mathcal{G} . For reductions of pure gravity one finds $\mathcal{G} = \mathcal{G}^{\text{isom}}$, but reductions of more general field contents can lead to bigger gauge groups $\mathcal{G} \supseteq \mathcal{G}^{\text{isom}}$.

Turning to the reduction over a 3D group manifold, the structure constants of the corresponding Lie algebra can be written as

$$f_{mn}{}^p = \epsilon_{mnq} Q^{pq} + 2\delta_{[m}{}^p a_{n]}. \quad (57)$$

By choosing an appropriate basis one can always take $Q^{pq} = \frac{1}{2} \text{diag}(q_1, q_2, q_3)$ with $q_i = 0, \pm 1$ and $a_m = (a, 0, 0)$ (see e.g. [58]). The generators T_m now satisfy

$$[T_1, T_2] = \frac{1}{2} q_3 T_3 - a T_2, \quad [T_2, T_3] = \frac{1}{2} q_1 T_1, \quad [T_3, T_1] = \frac{1}{2} q_2 T_2 + a T_3. \quad (58)$$

The Jacobi identity implies $q_1 a = 0$. This leads to 11 different algebras, given by the Bianchi classification [59, 60] in table 1. One distinguishes between algebras of class A with $f_{mn}{}^n = 0$ and class B with $f_{mn}{}^n \neq 0$. For the class B algebras we will use a notation that indicates that each class B algebra can be viewed as a deformation (with deformation parameter a) of a class A algebra, (see table 1).

Our starting point is 7D pure gravity $\hat{g}_{\hat{\mu}\hat{\nu}}$ with the Einstein–Hilbert term as Lagrangian. For dimensional reduction we split up indices in $\hat{\mu} = (\mu, m)$ where x^μ is the 4D spacetime while z^m corresponds to the 3D internal space. The reduction Ansatz from 7D to 4D reads:

$$\hat{ds}^2 = e^{-3\varphi/\sqrt{15}} g_{\mu\nu} dx^\mu dx^\nu + e^{2\varphi/\sqrt{15}} \mathcal{M}_{pq} U^p_m U^q_n dz^m dz^n, \quad (59)$$

where we have truncated away the three Kaluza–Klein vectors. The resulting 4D field content is $\{g_{\mu\nu}, \mathcal{M}_{mn}, \varphi\}$, where \mathcal{M}_{mn} parametrizes the coset $SL(3, \mathbb{R})/SO(3)$ and contains two dilatons and three axions (see [58, 61] for an explicit representation in terms of the five scalars). In all cases but one we will consider the following truncated parametrization of the scalar coset:

$$\mathcal{M}_{mn} = \text{diag}(e^{-2\sigma/\sqrt{3}}, e^{-\phi+\sigma/\sqrt{3}}, e^{\phi+\sigma/\sqrt{3}}), \quad (60)$$

where we have set the axions equal to zero¹¹. The matrix $U^m_n(z^p)$ contains the only dependence on the internal manifold [58]:

$$U^m_n = \begin{pmatrix} 1 & 0 & -s_{1,3,2} \\ 0 & e^{az^1} c_{2,3,1} & e^{az^1} c_{1,3,2} s_{2,3,1} \\ 0 & -e^{az^1} s_{3,2,1} & e^{az^1} c_{1,3,2} c_{2,3,1} \end{pmatrix}, \quad (61)$$

where we have used the following abbreviations:

$$c_{m,n,p} \equiv \cos\left(\frac{1}{2}\sqrt{q_m}\sqrt{q_n}z^p\right), \quad s_{m,n,p} \equiv \sqrt{q_m} \sin\left(\frac{1}{2}\sqrt{q_m}\sqrt{q_n}z^p\right)/\sqrt{q_n}, \quad (62)$$

and it is understood that the structure constants satisfy the Jacobi identity, amounting to $q_1 a = 0$. The U^m_n parametrize the different group manifolds and consist of the components of the Maurer–Cartan 1-forms. After reduction the U^m_n only appear in the z^m -independent combination:

$$f_{mn}{}^p = -2(U^{-1})^r{}_m (U^{-1})^s{}_n \partial_{[r} U^p{}_{s]}, \quad (63)$$

where $f_{mn}{}^p$ are the structure constants (57).

Before diving into the dimensional reduction we would like to address the issue of consistent truncations. A truncation is considered to be consistent if any solution to the truncated field equations leads to a solution of the full field equations [61, 62]. All truncations we consider in this section are consistent in this sense.

The reduction over a class A group manifold results in the following Lagrangian: [61]

$$\mathcal{L}_{\text{class A}} = \sqrt{-g} \left[R - \frac{1}{2}(\partial\vec{\phi})^2 + \frac{1}{2} e^{-\sqrt{5}\varphi/\sqrt{3}} \{[\text{Tr}(\mathcal{M}\mathcal{Q})]^2 - 2\text{Tr}(\mathcal{M}\mathcal{Q}\mathcal{M}\mathcal{Q})\} \right], \quad (64)$$

with $\vec{\phi} = (\varphi, \sigma, \phi)$. The scalar potential is always of multiple exponential form in the truncation (60). In table 2 we list the different possibilities to truncate the general scalar potential of class A. We will elaborate on these truncations in the next subsections. Note that the group manifolds generically gain isometries in these truncations; we also give the resulting manifold.

The reduction over a class B group manifold gives rise to a less familiar situation: one cannot even write a Lagrangian, but only field equations that cannot be derived as Euler–Lagrange equations [61]. From the massive deformations of the Einstein equations one can find a generalized class B-version of the potential of (64), including a -dependent terms, but the deformations of the scalar field equations are not the derivatives of this potential. However,

¹¹ This will be a consistent truncation if $a(q_2 e^{-\phi} - q_3 e^{+\phi}) = 0$, which is satisfied for all class A types ($a = 0$), class B type V ($q_2 = q_3 = 0$) and type VII_a (for $\phi = 0$).

Table 2. The different truncations of the class A group manifolds leading to multiple exponential potentials. The table also gives the corresponding internal manifold and its number of isometries. Here \times and \bowtie denote the direct and twisted products, respectively (see also section 3.3). The coset manifolds S^n and \mathbb{H}^n are understood to have the maximally symmetric metric. The cases with 4 and 6 isometries of type IX are called the squashed and round S^3 , respectively.

Bianchi	Truncation (60)	Potential	Isometries	Manifold
IX	–	6-tuple	3	$so(3)$
	$\phi = 0$	Double	4	$S^2 \bowtie S^1$
	$\phi = \sigma = 0$	Single	6	S^3
VIII	–	6-tuple	3	$so(2, 1)$
	$\phi = 0$	Double	4	$\mathbb{H}^2 \bowtie S^1$
VII ₀	–	Triple	3	$iso(2)$
	$\phi = 0$	–	6	\mathbb{E}^3
VI ₀	–	Triple	3	$iso(1, 1)$
	$\phi = 0$	Single	3	$iso(1, 1)$
II	–	Single	4	$\mathbb{E}^2 \bowtie S^1$
I	–	–	6	\mathbb{E}^3

we will encounter truncations for which the construction of a Lagrangian is possible. In these cases the truncation ensures that the massive deformations of the scalar field equations indeed are the derivatives of the potential derived from the Einstein equations.

3.2. Single exponential potentials

We first consider the possible truncations of the group manifold reductions to the system (1) with a single exponential potential:

$$\mathcal{L}_{\text{single}} = \sqrt{-g} \left[R - \frac{1}{2} (\partial \vec{\phi})^2 - \Lambda e^{\vec{\alpha} \cdot \vec{\phi}} \right]. \quad (65)$$

As explained in section 2.1 one can always choose a basis such that only one scalar appears in the potential. To avoid the need to do so we define the generalized parameter $\Delta = (\vec{\alpha} \cdot \vec{\alpha} - 3)_{\pm}$, which will be invariant under changes of basis. The subscript \pm denotes the sign of Δ .

The potential of class A is in general more complicated than the single exponential potential that we have considered in section 2. However, there are three different possibilities to truncate the potential given in (64) to (65). These are the type II, type VI₀ and type IX cases, as given in table 3. To be concrete we give the example of type II, whose Lagrangian with the truncation (60) reads

$$\mathcal{L} = \sqrt{-g} \left[R - \frac{1}{2} (\partial \varphi)^2 - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} (\partial \sigma)^2 - \frac{1}{8} e^{-\sqrt{3}\varphi/\sqrt{3+2\sigma}\sqrt{3+2\phi}} \right]. \quad (66)$$

Similar formulae can be derived from (64) in the other cases. The only possibility to obtain a negative potential of single exponential form occurs in the type IX potential. This corresponds to the gauging of $SO(3)$, which is the only compact gauge group of the Bianchi classification. Note that the truncation $\sigma = \phi = 0$, leading to a single exponential potential, corresponds to the case of maximal isometry enhancement such that the group manifold becomes the round S^3 (see table 2).

The full set of field equations of class B gaugings cannot be derived from a Lagrangian. However, for specific truncations this is possible. We know of three such cases leading to a

Table 3. The truncations to a single exponential potential of positive or negative sign and the reduction manifold corresponding to these truncations.

Bianchi	Class	Truncation	Δ	Manifold
II	A	(60)	4_+	$\mathbb{E}^2 \bowtie S^1$
VI ₀	A	(60), $\phi = 0$	0_+	$iso(1, 1)$
III	B	(67)	-1_+	$\mathbb{H}^2 \times S^1$
V	B	(60), $\sigma = \phi = 0$	$-4/3_+$	\mathbb{H}^3
VII _a	B	(60), $\sigma = \phi = 0$	$-4/3_+$	\mathbb{H}^3
IX	A	(60), $\sigma = \phi = 0$	$-4/3_-$	S^3

Lagrangian with a single exponential potential (see table 3). The truncation of type III is not of the form (60) but given by¹²:

$$\mathcal{M} = \begin{pmatrix} e^{-\sigma/\sqrt{3}} & 0 & 0 \\ 0 & e^{\sigma/2\sqrt{3}} \cosh(\frac{1}{2}\sqrt{3}\sigma) & -e^{\sigma/2\sqrt{3}} \sinh(\frac{1}{2}\sqrt{3}\sigma) \\ 0 & -e^{\sigma/2\sqrt{3}} \sinh(\frac{1}{2}\sqrt{3}\sigma) & e^{\sigma/2\sqrt{3}} \cosh(\frac{1}{2}\sqrt{3}\sigma) \end{pmatrix}. \quad (67)$$

In this truncation, the type III manifold becomes $\mathbb{H}^2 \times S^1$. Similarly, the truncations of type V and type VII_a correspond to the manifold \mathbb{H}^3 . Thus, in all three cases in which class B allows for an action, the group manifold (partly) reduces to a hyperbolic manifold. We furthermore see that in all cases a single exponential potential requires a group manifold with maximum isometry enhancement.

In summary, we find that the different group manifold reductions give rise to examples of the three classes of potentials of the form in (1) with Δ positive, zero or negative. In particular we find the following possibilities:

$$\Delta = -\frac{4}{3}, -1, 0, 4, \quad (68)$$

for positive potentials.

A very natural question concerns the uplift of the solutions in section 2 to 7D. By construction, all solutions with any of the Δ -values with subscript + listed in table 3 can be uplifted to a purely gravitational solution in 7D. Of these, the interpolating solution with $\Delta = -4/3$ uplifts to the fluxless limit of the S2-brane solution in 7D [12]. We will comment on the uplift of the interpolating solutions with $\Delta = -1, 0, 4$ in the next two subsections.

In addition, we have checked that the critical point solutions with $\Delta = -4/3, -1$ uplift to a 7D (locally) Minkowskian spacetime. This is similar to the results of [58], where an 8D cosmological power-law solution uplifted (via \mathbb{H}^3) to 11D (locally) Minkowskian spacetime. In fact, the $\Delta = -4/3$ case can be understood as a T^4 reduction of this (see also section 3.5). The $\Delta = -1$ case will have an analogous power-law solution in 8D and 9D, which will also uplift (via $\mathbb{H}^2 \times S^1$ and \mathbb{H}^2 , respectively) to 11D (locally) Minkowskian spacetime.

The above results can be understood in terms of the Milne parametrization of flat spacetime. In this coordinate system the spatial part of Minkowski spacetime is a hyperbolic space. Adding a three-dimensional flat space to an $(n+1)$ -dimensional Milne spacetime, one can reduce over the hyperbolic part to a four-dimensional cosmological solution [53]. In fact, this is similar to the reduction of $D = 11$ Minkowski spacetime to the $\Delta = -4/3_-$ domain wall solution in eight dimensions [63]. Here flat spacetime is written in polar coordinates

¹² Note that the type III case does not satisfy the constraint of footnote 11. The off-diagonal components of \mathcal{M} (corresponding to non-zero axions) are a consequence of our basis choice for the structure constants. A simple $SO(2)$ -rotation brings \mathcal{M} to diagonal form but introduces off-diagonal components in \mathbf{Q} .

Table 4. The different truncations to double exponential potentials and the reduction manifold corresponding to these truncations.

Bianchi	Class	Truncation	Δ_1	Δ_2	$\alpha_1 \cdot \alpha_2$	Manifold
VIII	A	(60), $\phi = 0$	4_+	-1_+	3	$\mathbb{H}^2 \bowtie S^1$
IX	A	(60), $\phi = 0$	4_+	-1_-	3	$S^2 \bowtie S^1$

allowing for a reduction over an S^3 instead of a hyperbolic space. This is the domain wall analogue of the cosmological ‘Rome’ solution.

3.3. Double exponential potentials and S-branes

In this subsection we consider the truncation to double exponential potentials. Furthermore, we point out the relation between certain group manifold reductions and another method of dimensional reduction, namely over a maximally symmetric space with flux [64] and S-branes.

We first consider the generalization of the 4D system (9) to double exponential potentials. The generalized Lagrangian reads

$$\mathcal{L}_{\text{multi}} = \sqrt{-g} \left[R - \frac{1}{2} (\partial \vec{\phi})^2 - \sum_{i=1}^n \Lambda_i e^{\vec{\alpha}_i \cdot \vec{\phi}} \right], \quad (69)$$

with $n = 2$. Following section 2.1 we define $\Delta_i = (\vec{\alpha}_i \cdot \vec{\alpha}_i - 3)_\pm$, where the subscript \pm denotes the sign of Λ_i . The general class A potential (64) gives rise to two potentials of this form (see table 4). The coefficients Λ_i can be taken to have the same absolute value. Due to the opposite signs of the exponentials in the type IX case, one of the scalars has a minimum and indeed this case can be truncated to a single exponential potential (with a negative sign, see table 3). We know of no class B truncations to a double exponential potential.

The reduction schemes leading to these double potentials can also be seen from another point of view. Due to the independence of the group manifold Ansatz (61) of z^3 , one can first perform a trivial reduction to 6D. This results in the Lagrangian:

$$\mathcal{L}_6 = \sqrt{-\hat{g}} \left[\hat{R} - \frac{1}{2} (\partial \hat{\sigma})^2 - \frac{1}{4} e^{-\sqrt{5}\hat{\sigma}/\sqrt{2}} \hat{F}_2^2 \right], \quad (70)$$

where \hat{F}_2 is the field strength of the Kaluza–Klein vector. All Bianchi group manifold reductions thus correspond to a non-trivial reduction of this 6D system. A subset of those will correspond to reductions over a maximally symmetric 2D space with flux, having the following reduction Ansatz:

$$\hat{ds}_6^2 = e^{-\varphi/\sqrt{2}} ds_4^2 + e^{\varphi/\sqrt{2}} d\Sigma_k^2, \quad \hat{F}_2 = \sqrt{2} f \text{vol}(\Sigma_k), \quad \hat{\sigma} = \sigma, \quad (71)$$

where the parameter f is the flux through the internal 2D manifold, which has constant curvature $k = 0, \pm 1$: we take $R_{ab}(\Sigma_k) = \frac{1}{2} k g_{ab}$. This results in a lower-dimensional theory with a double exponential potential [13, 64]:

$$\mathcal{L}_4 = \sqrt{-g} \left[R - \frac{1}{2} (\partial \sigma)^2 - \frac{1}{2} (\partial \varphi)^2 - f^2 e^{-\sqrt{5}\sigma/\sqrt{2}-3\varphi/\sqrt{2}} + k e^{-\sqrt{2}\varphi} \right]. \quad (72)$$

For $f \neq 0$ and $k \neq 0$ this double exponential potential has the same characteristics as given in table 4 with the sign of Λ_2 given by the sign of $-k$. For either of the potential terms vanishing one finds one of the cases of table 3.

By comparison with tables 3 and 4 one can relate the group manifold reductions to flux reductions over maximally symmetric 2D spaces, i.e. S^2 , \mathbb{E}^2 or \mathbb{H}^2 . Perhaps the most striking relation is the equivalence between the type IX group manifold with the truncations (60) and $\phi = 0$ and the flux reduction with $f \neq 0$ and $k = +1$ [63]. This corresponds to the three-sphere

Table 5. The different truncations to triple exponential potentials and the reduction manifold corresponding to these truncations.

Bianchi	Class	Truncation	Δ_1	Δ_2	Δ_3	$\vec{\alpha}_1 \cdot \vec{\alpha}_2$	$\vec{\alpha}_1 \cdot \vec{\alpha}_3$	$\vec{\alpha}_2 \cdot \vec{\alpha}_3$	Manifold
VI ₀	A	(60)	4 ₊	0 ₊	4 ₊	3	-1	3	<i>iso</i> (1, 1)
VII ₀	A	(60)	4 ₊	0 ₋	4 ₊	3	-1	3	<i>iso</i> (2)

as the Hopf fibration over S^2 , denoted as $S^2 \bowtie S^1$ in table 2. Due to the truncation $\phi = 0$, one gains an extra isometry and therefore this reduction is over the squashed three-sphere (see also table 2). The further truncation to $\sigma = 0$ brings one to the round S^3 and single exponential potential of section 3.2.

Without the flux the 3D manifold becomes the direct product $S^2 \times S^1$, which gives rise to a single exponential potential with $\Delta = -1_-$ in table 3. However, this is not a group manifold and therefore reduction over it does not necessarily preserve supersymmetry¹³; the resulting potential will generically not be embeddable within a gauged supergravity.

Similarly, the type VIII group manifold with the truncations (60) and $\phi = 0$ is equivalent to the flux reduction with $f \neq 0$ and $k = -1$. Thus the $so(2, 1)$ group manifold can be seen as a fibration over \mathbb{H}^2 , denoted as $\mathbb{H}^2 \bowtie S^1$. Its fluxless limit yields the direct product $\mathbb{H}^2 \times S^1$, which is a type III group manifold and indeed yields a single exponential potential with $\Delta = -1_+$ in table 3.

The remaining case is the type II group manifold, which can be seen as a fibration over \mathbb{E}^2 , denoted as $\mathbb{E}^2 \bowtie S^1$. This yields a single exponential potential with $\Delta = 4_+$. Taking the zero flux limit leads to the direct product $\mathbb{E}^2 \times S^1$, which is a group manifold of type I, leading to a vanishing potential.

Having shown that the above double exponential potentials have a higher-dimensional origin in six or seven dimensions, it is interesting to uplift any cosmological solution. Solutions to the single and double exponential cases uplift to various S-branes in 6D [12, 13, 16, 30, 32, 48]. In particular, the interpolating solution with $\Delta = -1$ uplifts to the fluxless limit of the hyperbolic S2-brane in 6D. Again, the critical point solution with $\Delta = -1$ uplifts to a (locally) Minkowskian spacetime in 6D [53].

From the above dictionary, one can uplift all these solutions one step further: to a purely gravitational solution in 7D. It would be very interesting to identify which geometry the interpolating solutions correspond to.

3.4. Triple exponential potentials and exotic S-branes

We now turn to the truncations to triple exponential potentials. These will turn out to be related to reductions over a circle with different $SL(2, \mathbb{R})$ twists.

The truncations we are interested in have the Lagrangian (69) with $n = 3$. The class A potential (64) gives rise to two different triple exponential potentials, which are given in table 5 and explicitly read (the class A Lagrangian (64) with $q_1 = 0$):

$$\mathcal{L} = \sqrt{-g} \left[R - \frac{1}{2} (\partial \vec{\phi})^2 - \frac{1}{8} e^{-\sqrt{3}\varphi} (q_2 e^{-\phi} - q_3 e^{\phi})^2 \right]. \quad (73)$$

Both triple exponential potentials allow for the truncation $\phi = 0$. For the type VI₀ case with $q_2 = -q_3$ this results in a single exponential potential with $\Delta = 0_+$ (as given in table 3). For the type VII₀ case with $q_2 = q_3$ the terms cancel out and one is left without a potential. In this truncation, the type VII₀ group manifold reduces to the type I manifold, in complete analogy

¹³ Supersymmetry would require $f \sim k$ but we have $f = 0$ and $k = +1$.

to the degeneration of type VII_a to type V in the truncation $\phi = \sigma = 0$. As in the double exponential case, we know of no class B truncations to a triple exponential potential.

Similar to the case of double exponential potentials, the above triple exponential potentials can be obtained from another point of view, which in this case is five-dimensional. The group manifold Ansatz (61) with $q_1 = 0$ is independent of z^3 and z^2 . Therefore, one can always reduce to 5D where the intermediate Lagrangian reads:

$$\begin{aligned}\mathcal{L}_5 &= \sqrt{-\hat{g}} \left[\hat{R} - \frac{1}{2}(\partial\hat{\sigma})^2 - \frac{1}{2}(\partial\hat{\phi})^2 - \frac{1}{2}e^{2\hat{\phi}}(\partial\hat{\chi})^2 \right], \\ &= \sqrt{-\hat{g}} \left[\hat{R} - \frac{1}{2}(\partial\hat{\sigma})^2 + \frac{1}{4}\text{Tr}(\partial\hat{\mathcal{K}}\partial\hat{\mathcal{K}}^{-1}) \right],\end{aligned}\quad (74)$$

where we have truncated the two Kaluza–Klein vectors and defined:

$$\hat{\mathcal{K}} = e^{\hat{\phi}} \begin{pmatrix} e^{-2\hat{\phi}} + \hat{\chi}^2 & \hat{\chi} \\ \hat{\chi} & 1 \end{pmatrix}. \quad (75)$$

Thus every Bianchi group manifold reduction with $q_1 = 0$ corresponds to a reduction of this 5D system.

The 5D system (74) has a global symmetry $\hat{\mathcal{K}} \rightarrow \Omega\hat{\mathcal{K}}\Omega^T$ with $\Omega \in SL(2, \mathbb{R})$. In addition the 5D field equations are invariant under the \mathbb{R}^+ transformation $\hat{g}_{\hat{\mu}\hat{\nu}} \rightarrow \lambda\hat{g}_{\hat{\mu}\hat{\nu}}$. Note that this transformation scales the Lagrangian but leaves the equations of motion invariant.

Every generator of a global symmetry can be employed for a reduction over a circle with a twist [65]. Restricting this generator to $SL(2, \mathbb{R})$ gives rise to three distinct lower-dimensional theories, with different gauge groups and different potentials [66, 67]. We will focus on these three cases in this subsection. If one also includes the \mathbb{R}^+ generator one obtains a lower-dimensional theory with only field equations [68]. This corresponds to a reduction over a class B group manifold. However, we will not consider these here.

The reduction Ansatz with an $SL(2, \mathbb{R})$ twist reads [69]

$$\begin{aligned}ds_5^2 &= e^{-\varphi/\sqrt{3}} ds_4^2 + e^{2\varphi/\sqrt{3}} (dz^1)^2, & \hat{\sigma} &= \sigma, & \hat{\mathcal{K}} &= \Omega\mathcal{K}\Omega^T, \\ \Omega &= \exp \begin{pmatrix} 0 & q_2 z^1 \\ q_3 z^1 & 0 \end{pmatrix},\end{aligned}\quad (76)$$

with $\mathcal{K} = \text{diag}(e^{-\phi}, e^{\phi})$. Here the two parameters q_2 and q_3 distinguish between the three different subgroups of $SL(2, \mathbb{R})$:

$$(q_2, q_3) = \begin{cases} (0, 1) : \Omega \in \mathbb{R}, \\ (1, 1) : \Omega \in SO(2), \\ (-1, 1) : \Omega \in SO(1, 1)^+. \end{cases} \quad (77)$$

Indeed, this reduction results in (73) for the Bianchi types II, VI₀ and VII₀ with appropriate values of q_2 and q_3 . Thus, we have shown that reduction over these specific Bianchi types corresponds to a twisted reduction from 5D with one of the three subgroups of $SL(2, \mathbb{R})$. The use of the \mathbb{R} subgroup corresponds to the flux compactification with flux $d\chi$.

We would like to comment on the 5D point of view of the generation of the single exponential potential $\Delta = 0_+$. The group manifold Ansatz in the truncations (60) and $\phi = 0$ boils down to a 5D reduction Ansatz with vanishing axion and a dilaton which only takes values in the internal z^1 space. This can be seen as a flux compactification where the dilaton $\hat{\phi}$ takes the role of the gauge potential, leading to $\Delta = 0_+$.

Let us now discuss the uplift of solutions of the triple exponential potentials of types VI₀ and VII₀ and the single exponential potential of type II to five and seven dimensions. The type II solution of section 2 will uplift to an S2-brane in 5D, which has not been given in the literature¹⁴. Solutions to the triple exponential case will uplift to exotic S2-branes in 5D,

¹⁴ The standard formula for Sp -brane solutions in D dimensions breaks down for $p = D - 3$.

but such solutions have not yet been constructed. The fact that $D = 5$ is not relevant in this construction: the axion-dilaton-gravity system will allow three such $S(D - 3)$ -solutions in arbitrary D .

Clearly, one can uplift all these solutions to purely gravitational solutions in 7D and it would be very interesting to identify which geometry the different solutions correspond to.

3.5. Embedding in M-theory

As discussed, the reduction of 7D gravity over a group manifold gives rise to a gauge group and a scalar potential in 4D. The 7D gravity can be trivially embedded in ungauged 7D maximal supergravity, which in itself arises from a trivial T^4 reduction of 11D supergravity, the low-energy limit of M-theory. Therefore, the 4D systems under consideration have an origin in M-theory. Moreover, since a group manifold reduction does not break supersymmetry¹⁵, the 4D systems can be embedded in a gauged maximal supergravity. In addition, the generalization to more general field contents can lead to bigger gauge groups $\mathcal{G} \supseteq \mathcal{G}^{\text{isom}}$. The following example illustrates these points.

Reduction of 7D ungauged maximal supergravity over a class A group manifold leads to 4D gauged supergravities. The gauge group is $CSO(p, q, 8 - p - q)$ for p positive entries and q negative entries in the mass matrix¹⁶ Q^{mn} and therefore $p + q \leq 3$. The same theory can be obtained by a reduction over T^4 of the 8D $CSO(p, q, 3 - p - q)$ gauged maximal supergravity [61]. It can be viewed as a contracted version of the $SO(p, 8 - p)$ gauged maximal supergravity in 4D, therefore having a contracted gauge group and a different scalar potential. Note that this reduction provides an example of gauge symmetry enhancement: the dimension of the $CSO(p, q, 8 - p - q)$ gauge group is larger than three.

From the previous discussion it follows that the positive single exponential potentials of class A, with $\Delta = 0, 4$ can be embedded into gauged maximal supergravities in 4D with the following gauge groups:

$$\Delta = 0 : CSO(1, 1, 6), \quad \Delta = 4 : CSO(1, 0, 7). \quad (78)$$

We would also like to mention some results on other 4D gaugings, which are also of the CSO -form but not obtainable by a 3D group manifold reduction. The following truncations to positive single exponential potentials are possible [8, 70, 71]:

$$\Delta = -3 : CSO(4, 4, 0), CSO(5, 3, 0), \quad \Delta = -8/3 : CSO(3, 3, 2). \quad (79)$$

The latter is the reduction of the $SO(3, 3)$ gauged maximal supergravity in $D = 5$. These values for Δ correspond to pure cosmological constants, allowing for de Sitter solutions, in 4D and 5D respectively. In [8], it was already noted that the reduction of the 5D de Sitter solution to 4D yields an accelerating cosmology corresponding to the critical point solution with $\Delta = -8/3$. The corresponding interpolating solution of section 2 will uplift to a 5D solution which asymptotes to de Sitter.

Reduction of 7D maximal ungauged supergravity over a class B group manifold will also result in a 4D gauged maximal supergravity, which however lacks a Lagrangian. The same gauged supergravity can be obtained by a T^4 reduction of the class B gauged supergravities in 8D [58]. In analogy with table 1 we consider these gauged supergravities as deformations of class A with the deformation parameter a . Therefore, we will denote their gauge groups by $CSO(p, q, 8 - p - q)_a$ with $p + q \leq 2$ (note the stricter range of $p + q$ as compared to

¹⁵ At least when reducing over the universal covering space of group manifolds. When considering reductions over compactified versions, supersymmetry is not necessarily preserved (see section 4).

¹⁶ See the appendix in [61] for the relation between the Bianchi classification and $CSO(p, q, r)$ groups [70].

class A; this follows from the Jacobi identity). With this understanding one can assign the single exponential truncations of the class B results with $\Delta = -4/3, -1$ to the following maximal gauged supergravities:

$$\Delta = -\frac{4}{3} : CSO(0, 0, 8)_{a=1}, CSO(2, 0, 6)_a, \quad \Delta = -1 : CSO(1, 1, 6)_{a=1/2}. \quad (80)$$

4. Comments

We would like to comment on the relevance of the interpolating solutions to inflation. In this context the number of e -foldings is crucial. It is defined by $N_e = \log(a(\tau_2)/a(\tau_1))$ with τ_1 and τ_2 the start and end times of the accelerating period. These times can easily be found in our approach as the points where the straight lines intersect the ‘arctic circle’. The number of e -foldings is required to be of the order of 65 to account for astronomical data. For the interpolating solutions with $\Delta > -2$, which is a necessary requirement to have a finite period of acceleration, one finds $N_e \lesssim 1$ [14, 16, 17] for all values of a_1, a_2 and Δ . The only exception to this behaviour is when $\Delta \rightarrow -2$, where N_e blows up. For the required 65 e -foldings one needs to take $\Delta + 2 \sim 10^{-60}$. As an example, for a compactification over an m -dimensional hyperbolic space, leading to $\Delta = -2 + 2/m$, this translates into $m \sim 10^{60}$. Thus, it seems that the e -foldings requirement for inflation can be met by a single exponential potential with a higher-dimensional origin, be it admittedly tremendously higher and evidently not in the context of M- or string theory. For the present-day acceleration, however, solutions for exponential potentials from M-theory could be relevant.

In section 3 we have discussed reduction over the Bianchi group manifolds. There we have taken the universal cover of the group manifold. For this reason, types I–VIII are non-compact and have the topology of \mathbb{R}^3 while type IX has the topology of S^3 . Therefore, the latter case does not raise any issues when compactifying. In the case of non-compact group manifolds there are two approaches: (1) supersymmetry is preserved but one can only use the reduction to uplift solutions since the non-compact internal manifold leads to a continuous spectrum in the 4D theory, which is not physically acceptable. This is the so-called non-compactification scheme; (2) the group manifold is compactified by dividing out by discrete symmetries [72]. For all Bianchi types except types IV and VI_a it is possible to construct compact manifolds in this way [73]. Sometimes supersymmetry is preserved under this operation, like for the three-torus, but sometimes it is not, like for a hyperbolic manifold. In the latter case one clearly cannot embed the system (9) in a gauged supergravity.

Our analysis of the triple exponential potentials suggests the existence of exotic $S(D-3)$ -branes in D dimensions. The situation with one standard and two exotic solutions stemming from the different twisted $SL(2, \mathbb{R})$ reductions is analogous to the structure of domain wall solutions in these systems. This was analysed in the case of the reduction of IIB supergravity to 9D gauged maximal supergravity. The three distinct domain walls uplift to one standard and two exotic half-supersymmetric 7-brane solutions of IIB [67]. These carry charges that take values in one of the three inequivalent subgroups of $SL(2, \mathbb{R})$. The standard case is the D7-brane and corresponds to a charge vector in the \mathbb{R} subgroup of $SL(2, \mathbb{R})$. The exotic cases have charge vectors that take values in the other two subgroups of $SL(2, \mathbb{R})$ and can be interpreted as a D7-Q7 or a D7- $\overline{Q7}$ bound state [67], where the Q7 is the S-dual of the D7.

A similar situation arises with instantons where we also find solutions with charge vectors in all three subgroups of $SL(2, \mathbb{R})$ [74]. Note that these solve the same axion–dilaton–gravity system as the S-branes and the 7-branes, with the only difference being that in the case of the instantons we work with a Euclidean version. Again, the standard D-instanton has a charge vector that takes values in the \mathbb{R} subgroup of $SL(2, \mathbb{R})$. The interpretation of the exotic

instanton-like solutions corresponding to the other two subgroups of $SL(2, \mathbb{R})$ is less clear [74].

In the case of the S-branes it would be interesting to see whether, analogously to the 7-branes and instantons, solutions corresponding to each of the three subgroups can be constructed. This would mean the existence of new ‘exotic’ S-branes, which have not been given in the literature.

Finally, it would be interesting to extend our work to more general potentials with multiple scalars. It has been argued that multi-exponential potentials with orthogonal dilaton couplings lead to assisted inflation for the power-law solutions [34]. It would be very interesting to investigate the consequences for the interpolating solutions as well. Other simplifications might occur for the case when the dilaton couplings form the Cartan matrix of a semi-simple Lie group; in such cases the system becomes an (integrable) Toda model [43, 47, 75].

Acknowledgments

We thank Marc Henneaux and Román Linares for interesting discussions. This work is supported in part by the European Community’s Human Potential Programme under contract HPRN-CT-2000-00131 Quantum Spacetime, in which EB, AC, MN and DR are associated with Utrecht University. The work of UG is funded by the Swedish Research Council.

References

- [1] Riess A G *et al* (*Supernova Search Team Collaboration*) 1998 Observational evidence from supernovae for an accelerating universe and a cosmological constant *Astron. J.* **116** 1009–38 (*Preprint astro-ph/9805201*)
- [2] Perlmutter S *et al* (*Supernova Cosmology Project Collaboration*) 1999 Measurements of omega and lambda from 42 high-redshift supernovae *Astrophys. J.* **517** 565–86 (*Preprint astro-ph/9812133*)
- [3] Kallosh R, Linde A D, Prokushkin S and Shmakova M 2002 Gauged supergravities, de Sitter space and cosmology *Phys. Rev. D* **65** 105016 (*Preprint hep-th/0110089*)
- [4] Frè P, Trigiante M and Proeyen A Van 2002 Stable de Sitter vacua from $N = 2$ supergravity *Class. Quantum Grav.* **19** 4167–94 (*Preprint hep-th/0205119*)
- [5] Roo M de, Westra D B and Panda S 2003 de Sitter solutions in $N = 4$ matter coupled supergravity *J. High Energy Phys.* JHEP02(2003)003 (*Preprint hep-th/0212216*)
- [6] Kachru S, Kallosh R, Linde A and Trivedi S P 2003 de Sitter vacua in string theory *Phys. Rev. D* **68** 046005 (*Preprint hep-th/0301240*)
- [7] Roo M de, Panda S, Trigiante M and Westra D B 2003 Potential and mass-matrix in gauged $N = 4$ supergravity (*Preprint hep-th/0310187*)
- [8] Townsend P K 2001 Quintessence from M-theory *J. High Energy Phys.* JHEP11(2001)042 (*Preprint hep-th/0110072*)
- [9] Kallosh R and Linde A 2003 M-theory, cosmological constant and anthropic principle *Phys. Rev. D* **67** 023510 (*Preprint hep-th/0208157*)
- [10] Ohta N 2003 Intersection rules for S-branes *Phys. Lett. B* **558** 213–20 (*Preprint hep-th/0301095*)
- [11] Townsend P K and Wohlfarth M N R 2003 Accelerating cosmologies from compactification *Phys. Rev. Lett.* **91** 061302 (*Preprint hep-th/0303097*)
- [12] Ohta N 2003 Accelerating cosmologies from S-branes *Phys. Rev. Lett.* **91** 061303 (*Preprint hep-th/0303238*)
- [13] Roy S 2003 Accelerating cosmologies from M/string theory compactifications *Phys. Lett. B* **567** 322–9 (*Preprint hep-th/0304084*)
- [14] Wohlfarth M N R 2003 Accelerating cosmologies and a phase transition in M-theory *Phys. Lett. B* **563** 1 (*Preprint hep-th/0304089*)
- [15] Emparan R and Garriga J 2003 A note on accelerating cosmologies from compactifications and S-branes *J. High Energy Phys.* JHEP05(2003)028 (*Preprint hep-th/0304124*)
- [16] Ohta N 2003 A study of accelerating cosmologies from superstring / M theories *Prog. Theor. Phys.* **110** 269–83 (*Preprint hep-th/0304172*)
- [17] Chen C-M, Ho P-M, Neupane I P and Wang J E 2003 A note on acceleration from product space compactification *J. High Energy Phys.* JHEP07(2003)017 (*Preprint hep-th/0304177*)

- [18] Gutperle M, Kallosh R and Linde A 2003 M / string theory, S-branes and accelerating universe *J. Cosmol. Astroparts. Phys.* JCAP07(2003)001 (*Preprint hep-th/0304225*)
- [19] Chen C-M, Ho P-M, Neupane I P, Ohta N and Wang J E 2003 Hyperbolic space cosmologies *J. High Energy Phys.* JHEP10(2003)058 (*Preprint hep-th/0306291*)
- [20] Wohlfarth M N R 2003 Inflationary cosmologies from compactification *Preprint hep-th/0307179*
- [21] Townsend P K 2003 Cosmic acceleration and M-theory *Preprint hep-th/0308149*
- [22] Neupane I P 2003 Inflation from string/M-theory compactification? *Preprint hep-th/0309139*
- [23] Järv L, Mohaupt T and Saueressig F 2003 M-theory cosmologies from singular Calabi–Yau compactifications *Preprint hep-th/0310174*
- [24] Neupane I P 2003 Accelerating cosmologies from exponential potentials *Preprint hep-th/0311071*
- [25] Vieira P G 2003 Late-time cosmic dynamics from M-theory *Preprint hep-th/0311173*
- [26] Demaret J, Hanquin J L, Henneaux M and Spindel P 1985 Cosmological models in eleven-dimensional supergravity *Nucl. Phys. B* **252** 538–60
- [27] Halliwell J J 1987 Scalar fields in cosmology with an exponential potential *Phys. Lett. B* **185** 341
- [28] Burd A B and Barrow J D 1988 Inflationary models with exponential potentials *Nucl. Phys. B* **308** 929–45
- [29] Aguirregabiria J M, Feinstein A and Ibáñez J 1993 Exponential potential scalar field universes. 1 The Bianchi I models *Phys. Rev. D* **48** 4662–8 (*Preprint gr-qc/9309013*)
- [30] Lü H, Mukherji S, Pope C N and Xu K W 1997 Cosmological solutions in string theories *Phys. Rev. D* **55** 7926–35 (*Preprint hep-th/9610107*)
- [31] Poppe R and Schwager S 1997 String Kaluza–Klein cosmologies with RR-fields *Phys. Lett. B* **393** 51–8 (*Preprint hep-th/9610166*)
- [32] Lü H, Mukherji S and Pope C N 1999 From p-branes to cosmology *Int. J. Mod. Phys. A* **14** 4121–42 (*Preprint hep-th/9612224*)
- [33] Coley A A, Ibáñez J and van den Hoogen R J 1997 Homogeneous scalar field cosmologies with an exponential potential *J. Math. Phys.* **38** 5256–71
- [34] Liddle A R, Mazumdar A and Schunck F E 1998 Assisted inflation *Phys. Rev. D* **58** 061301 (*Preprint astro-ph/9804177*)
- [35] Malik K A and Wands D 1999 Dynamics of assisted inflation *Phys. Rev. D* **59** 123501 (*Preprint astro-ph/9812204*)
- [36] Copeland E J, Mazumdar A and Nunes N J 1999 Generalized assisted inflation *Phys. Rev. D* **60** 083506 (*Preprint astro-ph/9904309*)
- [37] Green A M and Lidsey J E 2000 Assisted dynamics of multi-scalar field cosmologies *Phys. Rev. D* **61** 067301 (*Preprint astro-ph/9907223*)
- [38] Billyard A P, Coley A A, Lidsey J E and Nilsson U S 2000 Dynamics of M-theory cosmology *Phys. Rev. D* **61** 043504 (*Preprint hep-th/9908102*)
- [39] Coley A A and van den Hoogen R J 2000 The dynamics of multi-scalar field cosmological models and assisted inflation *Phys. Rev. D* **62** 023517 (*Preprint gr-qc/9911075*)
- [40] Aguirregabiria J M, Chamorro A, Chimento L P and Zuccalá N A 2000 Assisted inflation in Friedmann–Robertson–Walker and Bianchi spacetimes *Phys. Rev. D* **62** 084029 (*Preprint gr-qc/0006108*)
- [41] Heard I P C and Wands D 2002 Cosmology with positive and negative exponential potentials *Class. Quantum Grav.* **19** 5435–48 (*Preprint gr-qc/0206085*)
- [42] Guo Z-K, Piao Y-S, Cai R-G and Zhang Y-Z 2003 Cosmological scaling solutions and cross-coupling exponential potential *Preprint hep-th/0306245*
- [43] Gavrilov V R, Ivashchuk V D and Melnikov V N 1994 Multidimensional cosmology with multicomponent perfect fluid and Toda lattices *Preprint gr-qc/9407019*
- [44] Cornalba L and Costa M S 2002 A new cosmological scenario in string theory *Phys. Rev. D* **66** 066001 (*Preprint hep-th/0203031*)
- [45] Ivashchuk V D and Melnikov V N 1989 Perfect fluid type solution in multidimensional cosmology *Phys. Lett. A* **136** 465–7
- [46] Bleyer U and Zhuk A 1994 Multidimensional integrable cosmological models with dynamical and spontaneous compactification *Preprint gr-qc/9405028*
- [47] Lukas A, Ovrut B A and Waldram D 1997 String and M-theory cosmological solutions with Ramond forms *Nucl. Phys. B* **495** 365–99 (*Preprint hep-th/9610238*)
- [48] Ivashchuk V D and Kim S W 2000 Solutions with intersecting p-branes related to Toda chains *J. Math. Phys.* **41** 444–60 (*Preprint hep-th/9907019*)
- [49] Gutperle M and Strominger A 2002 Spacelike branes *J. High Energy Phys.* JHEP04(2002)018 (*Preprint hep-th/0202210*)
- [50] Chen C-M, Gal'tsov D V and Gutperle M 2002 S-brane solutions in supergravity theories *Phys. Rev. D* **66** 024043 (*Preprint hep-th/0204071*)

- [51] Kruczenski M, Myers R C and Peet A W 2002 Supergravity S-branes *J. High Energy Phys.* JHEP05(2002)039 (Preprint hep-th/0204144)
- [52] Deger N S and Kaya A 2002 Intersecting S-brane solutions of $D = 11$ supergravity *J. High Energy Phys.* JHEP07(2002)038 (Preprint hep-th/0206057)
- [53] Russo J G 2003 Cosmological string models from Milne spaces and $SL(2, \mathbb{Z})$ orbifold Preprint hep-th/0305032
- [54] Lü H, Pope C N, Sezgin E and Stelle K S 1995 Stainless super p-branes *Nucl. Phys. B* **456** 669–98 (Preprint hep-th/9508042)
- [55] Copeland E J, Liddle A R and Wands D 1998 Exponential potentials and cosmological scaling solutions *Phys. Rev. D* **57** 4686–90 (Preprint gr-qc/9711068)
- [56] Cornalba L and Costa M S 2003 Time-dependent orbifolds and string cosmology Preprint hep-th/0310099
- [57] Scherk J and Schwarz J H 1979 How to get masses from extra dimensions *Nucl. Phys. B* **153** 61–88
- [58] Bergshoeff E, Gran U, Linares R, Nielsen M, Ortín T and Roest D 2003 The Bianchi classification of maximal $D = 8$ gauged supergravities *Class. Quantum Grav.* **20** 3997–4014 (Preprint hep-th/0306179)
- [59] Bianchi L 1898 Sugli spazi a tre dimensioni che ammettono un gruppo continuo di movimenti *Mem. Mat. Fis. Soc. Ital. Sci. Serie Terza* **11** 267–352
- Bianchi L 2001 On the three-dimensional spaces which admit a continuous group of motions *Gen. Rel. Grav.* **33** 2171–253 (Engl. Transl.)
- Jantzen R T 2001 Editor's note on On the three-dimensional spaces which admit a continuous group of motions by Luigi Bianchi *Gen. Rel. Grav.* **33** 2157–70 (Engl. Transl.)
- [60] Ellis G F R and MacCallum M A H 1969 A class of homogeneous cosmological models *Commun. Math. Phys.* **12** 108
- [61] Alonso Alberca N, Bergshoeff E, Gran U, Linares R, Ortín T and Roest D 2003 Domain walls of $D = 8$ gauged supergravities and their $D = 11$ origin *J. High Energy Phys.* JHEP06(2003)038 (Preprint hep-th/0303113)
- [62] Cvetič M, Gibbons G W, Lü H and Pope C N 2003 Consistent group and coset reductions of the bosonic string *Class. Quantum Grav.* **20** 5161–94 (Preprint hep-th/0306043)
- [63] Boonstra H J, Skenderis K and Townsend P K 1999 The domain wall/QFT correspondence *J. High Energy Phys.* JHEP01(1999)003 (Preprint hep-th/9807137)
- [64] Bremer M S, Duff M J, Lü H, Pope C N and Stelle K S 1999 Instanton cosmology and domain walls from M -theory and string theory *Nucl. Phys. B* **543** 321–64 (Preprint hep-th/9807051)
- [65] Scherk J and Schwarz J H 1979 Spontaneous breaking of supersymmetry through dimensional reduction *Phys. Lett. B* **82** 60
- [66] Hull C M 2002 Gauged $D = 9$ supergravities and Scherk–Schwarz reduction Preprint hep-th/0203146
- [67] Bergshoeff E, Gran U and Roest D 2002 Type IIB seven-brane solutions from nine-dimensional domain walls *Class. Quantum Grav.* **19** 4207–26 (Preprint hep-th/0203202)
- [68] Bergshoeff E, Wit de T, Gran U, Linares R and Roest D 2002 (Non-)Abelian gauged supergravities in nine dimensions *J. High Energy Phys.* JHEP10(2002)061 (Preprint hep-th/0209205)
- [69] Meessen P and Ortín T 1999 An $SL(2, \mathbb{Z})$ multiplet of nine-dimensional type II supergravity theories *Nucl. Phys. B* **541** 195–245 (Preprint hep-th/9806120)
- [70] Hull C M 1985 The minimal couplings and scalar potentials of the gauged $N=8$ supergravities *Class. Quantum Grav.* **2** 343
- [71] Günaydin M, Romans L J and Warner N P 1986 Compact and noncompact gauged supergravity theories in five-dimensions *Nucl. Phys. B* **272** 598
- [72] Thurston W P 1979 *The Geometry and Topology of 3-Manifolds* (Princeton, NJ: Princeton University press)
- [73] Barrow J and Kodama H 2001 The Isotropy of Compact Universes *Class. Quantum Grav.* **18** 1753–66 (Preprint gr-qc/0012075)
- [74] Bergshoeff E, Collinucci A, Gran U, Roest D and Vandoren S, in preparation
- [75] Lü H and Pope C N 1997 $SL(N+1, \mathbb{R})$ Toda solitons in supergravities *Int. J. Mod. Phys. A* **12** 2061–74 (Preprint hep-th/9607027)